ANTENNA SPHERICAL COORDINATE SYSTEMS AND THEIR APPLICATION IN COMBINING RESULTS FROM DIFFERENT ANTENNA ORIENTATIONS

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ABSTRACT

The results of theoretical calculations or measurements on antennas are generally given in terms of the vector components of the radiated electric field as a function of direction or position. Both the vector components and the direction parameters must be defined with respect to a spherical coordinate system fixed to the antenna. Along the principal planes there is no ambiguity about the terms such as vertical or horizontal component, but off the principal planes the definition of directions and vector components depends on how the spherical coordinate system is defined. This paper will define four different spherical coordinates that are commonly used in measurements and calculations, suggest a terminology that is useful to distinguish between them, and define the mathematical transformations between them. One important application of these concepts arises when comparing or combining measurement results from two antenna orientations. In this case, the axis of rotation dictates the preferred coordinate system and vector components. Measured results will be shown to illustrate the proper choice of coordinates for each situation.

1. INTRODUCTION

An antenna coordinate system is implicit in almost every antenna measurement. Terms such as pattern, main and cross-component, beam pointing and peak gain, imply the definition of directions and/or vector field components which require a coordinate system. Reference is often made to **the** cross-component of an antenna as if there is a unique definition of such a quantity when in fact the cross-component as well as the main component will depend on which coordinate system is used and how it is oriented with respect to the antenna. Ambiguity and confusion can be avoided by using precise definitions and a terminology that distinguishes the different coordinates. Ludwig[1] proposed such a terminology, and it is widely used. It does not clearly define all the coordinate systems commonly used however, and the following discussion will propose additions to the Ludwig terminology.

2.THETA PHI SPHERICAL COORDINATES

We begin by defining the X-Y-Z axes of the antenna coordinate system such that the main beam is approximately along the Z-axis. The X- or Y-axes are defined approximately coincident with the major polarization axis as shown in Figure 1. The precise definition of the axes location and orientation must be accomplished by using fiducial marks, alignment mirrors or optical telescopes. Once this definition is accomplished, the X- Y- and Z-axes remain fixed to the antenna and do not change for any of the spherical coordinate systems that will be discussed.

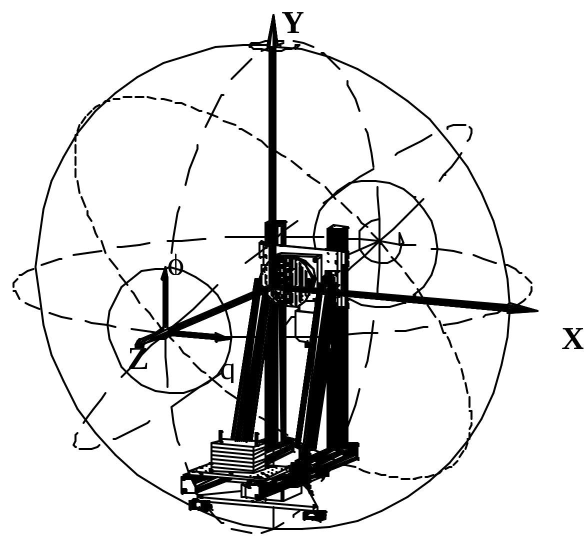

Figure 1 q-f **Spherical Coordinates**

The spherical coordinates shown in Figure 1 are the usual θ-ϕ coordinates with the Z-axis as the polar axis. The sphere surrounding the antenna should be much larger than the antenna since it represents the sphere on which the far-field vector components are measured or represented. For convenience it is shown just enclosing the antenna. With this coordinate system, directions are specified by the angles (θ,ϕ) and vector components by the unit vectors $\underline{q}$ and $\underline{f}$. The rotator system used for this coordinate system is the roll over azimuth positioner shown in Figure 1. In a far–field measurement, a vertically polarized source antenna illuminates the AUT with a field that is polarized in the $\underline{f}$-direction and the received signal will then correspond to the ϕ-component pattern. A horizontally polarized source will produce the θ-component pattern. It is important to note that when the antenna is rotated, the sphere defining the spherical coordinates and components stays fixed to the AUT and also rotates. The sphere is used to define the pattern radiated by the AUT as a function of coordinates fixed to the antenna. This sphere is not used to specify the directions the main beam points relative to axes fixed in space as the angles are changed.

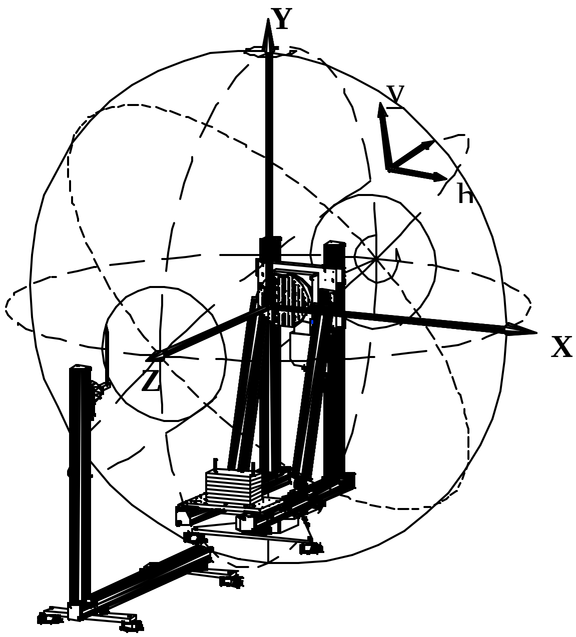

Figure 2 Ludwig-3 or h-v vector components.

If the AUT is linearly polarized, the vectors q and f shown in Figure 1 are not the most appropriate for specifying the field vectors since in the region of the main beam the vectors change direction relative to the antenna as a function of φ. The orientation of the Z-axis relative to the antenna could be changed to define new spherical coordinates, but there are some situations where the current definition is preferable, and it also makes it much easier to transform between the other spherical coordinates if we retain this definition for the q f spherical coordinates.

3. LUDWIG-3 OR H-V VECTOR COMPONENTS

A modification of the θ-φ components overcomes the polarization problem noted above, and is widely used in anechoic chamber measurements. The q and f unit vectors are rotated about the radial direction by the angle φ to obtain the vector components referred to as Ludwig-3 components. We have used the notation **h** and **v** to refer to these components since they define vectors that are respectively approximately horizontal (parallel to the x-axis) and vertical (parallel to the y-axis) over most of the hemisphere. The resulting coordinates and vectors for this coordinate system are shown in Figure 2. In a far-field measurement using the Ludwig-3 components, the source antenna is rotated about its axis by the angle φ in synchronization with the AUT φ-rotation.

If the magnitude and phase of two components in one system are known, the components in the other one are found from the following transformations.

$$E_h(q,f) = E_q(q,f)\cos f - E_f(q,f)\sin f$$
$$E_v(q,f) = E_q(q,f)\sin f + E_f(q,f)\cos f \qquad (1)$$

Corresponding transformations will be shown for all the coordinate systems.

4.LUDWIG-2, AZIMUTH-ELEVATION COMPONENTS

There are actually two coordinate systems that come under the Ludwig-2 definition. They correspond to different types of far-field rotators and the corresponding orientations of the polar axis of the spherical coordinates. The A-E components are used with the Azimuth over Elevation rotator shown in Figure 3 where the polar axis is coincident with the Y-axis.

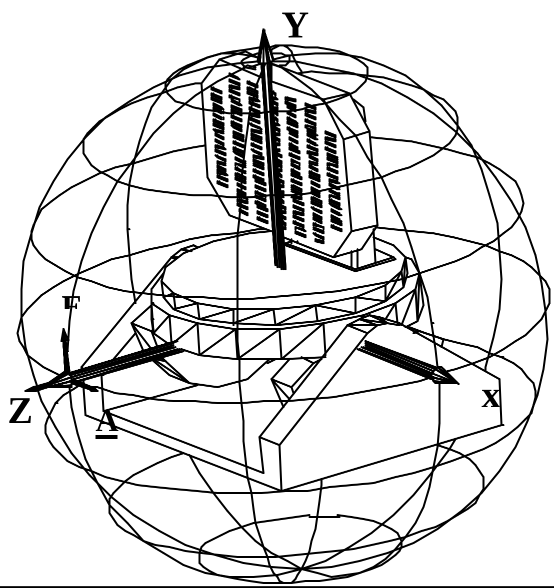

Figure 3 Az-El Coordinate system and azimuth over elevation rotator.

With this rotator, the origin of the AUT coordinate system is not centered on the antenna. It is at the intersection of the two axes of rotation, which is inside the mechanical structure. This will not effect the far-field amplitude pattern, but will change the phase pattern.

The rotation of the sphere with the AUT for this coordinate system is apparent in Figure 4 where the AUT has been rotated in both Azimuth and Elevation. The point on the sphere with coordinates (A,E) is now along the line from the origin to the source antenna, and the **A** and **E** components at this location will be measured by horizontally and vertically polarized source antennas respectively.

The transformations between θ-φ components and A-E components is given by,

$$E_A(A,E) = \frac{\cos\phi}{\cos E} E_\theta(\theta,\phi) - \frac{\cos\theta \sin\phi}{\cos E} E_\phi(\theta,\phi)$$

(2)

$$E_E(A,E) = \frac{\cos\theta \sin\phi}{\cos E} E_\theta(\theta,\phi) + \frac{\cos\phi}{\cos E} E_\phi(\theta,\phi)$$

where

$$\cos E = \sqrt{1-(\sin\theta \sin\phi)^2} \ . \qquad (3)$$

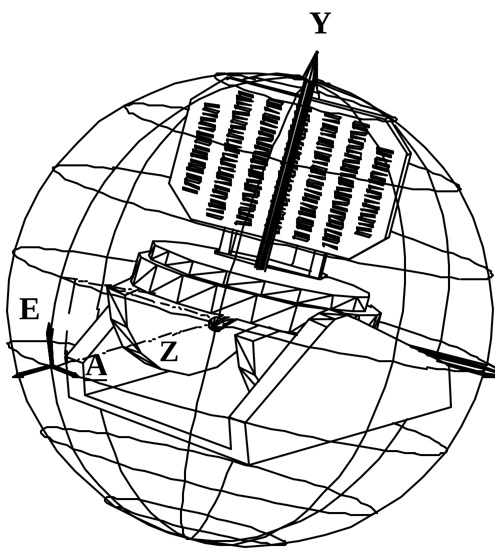

Figure 4 Rotated azimuth over elevation rotator.

5. LUDWIG-2 ALPHA-EPSILON COMPONENTS

The other Ludwig-2 components are associated with an elevation over azimuth rotator where the polar axis is coincident with the X-axis shown in Figure 5.

The transformations for these components are,

$$\mathrm{E}_\alpha(\alpha,\varepsilon) = \frac{\cos\theta \cos\phi}{\cos\alpha} \mathrm{E}_\theta(\theta,\phi) - \frac{\sin\phi}{\cos\alpha} \mathrm{E}_\phi(\theta,\phi)$$

(4)

$$\mathrm{E}_\varepsilon(\alpha,\varepsilon) = \frac{\sin\phi}{\cos\alpha} \mathrm{E}_\theta(\theta,\phi) + \frac{\cos\theta \cos\phi}{\cos\alpha} \mathrm{E}_\phi(\theta,\phi)$$

$$\mathrm{E}_\alpha(\alpha,\varepsilon) = \frac{\cos A}{\cos\alpha} \mathrm{E}_A(A,E) - \frac{\sin A Sin E}{\cos\alpha} \mathrm{E}_E(A,E)$$

(5)

$$\mathrm{E}_\varepsilon(\alpha,\varepsilon) = \frac{\sin A Sin E}{\cos\alpha} \mathrm{E}_A(A,E) + \frac{\cos A}{\cos\alpha} \mathrm{E}_E(A,E)$$

The angles in the different coordinate systems are related by,

$$\begin{aligned} \sin\theta \cos\phi &= \cos E \sin A = \sin\alpha, \\ \sin\theta \sin\phi &= \sin E = \cos\alpha \sin\varepsilon, \\ \cos\theta &= \cos E \cos A = \cos\alpha \cos\varepsilon. \end{aligned} \qquad (6)$$

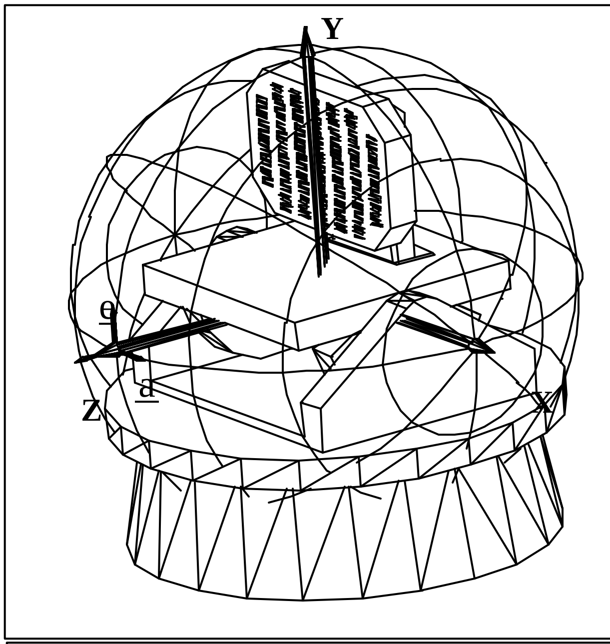

Figure 5 α-ε **coordinate system with associated elevation over azimuth rotator.**

Along the xz principal plane,

$$\mathrm{E}_A(A,0) = \mathrm{E}_\alpha(\alpha,0) = \mathrm{E}_h(\theta, 0\, or\, \pi) = \pm \mathrm{E}_\theta(\theta, 0\, or\, \pi)$$

(7)

$$\mathrm{E}_E(A,0) = \mathrm{E}_\varepsilon(\alpha,0) = \mathrm{E}_v(\theta, 0\, or\, \pi) = \pm \mathrm{E}_\phi(\theta, 0\, or\, \pi)$$

And along the yz-principal plane,

$$\mathrm{E}_A(0,E) = \mathrm{E}_\alpha(0,\varepsilon) = \mathrm{E}_h(\theta, \pm\frac{\pi}{2}) = \mp \mathrm{E}_\theta(\theta, \pm\frac{\pi}{2})$$

(8)

$$\mathrm{E}_E(0,E) = \mathrm{E}_\varepsilon(0,\varepsilon) = \mathrm{E}_v(\theta, \pm\frac{\pi}{2}) = \pm \mathrm{E}_\theta(\theta, \pm\frac{\pi}{2})$$

Therefore along the principal planes, there is little or no difference between the components, but as we move off the principal planes, the differences increase. This means for example that if a far-field measurement using an Elevation over Azimuth rotator is compared to the results of a near-field measurement, the comparison will depend on which components are used in the near-field program. All the components will show good correlation along the principal planes, however in directions off the principal planes, only the α-ε components will show good agreement with the EL/AZ far-field measurements. If measured results from near-field, far-field or anechoic measurements are compared to theoretical predictions, the vector components must be the same for measurement and theory or the results will not agree. Such disagreement can mistakenly be interpreted as indication of problems with measurements or antenna construction when the actual cause is an inconsistent choice of coordinate systems.

6. COMBINING RESULTS FROM DIFFERENT ANTENNA ORIENTATIONS

Another measurement application that requires the proper choice of both the vector components and the coordinate system used to define directions has recently been developed at Nearfield Systems Inc. This application uses two or more planar near-field measurements on the same antenna where the antenna is rotated about one axis between each measurement. The results of these measurements can be used to extend the angular coverage of the planar near-field measurements or to estimate the magnitude of specific errors in a measurement. Both of these applications will be illustrated.

Figure 6 Slot Array test antenna in initial un-rotated orientation.

The angular coverage of planar near-field measurements is limited by the size of the scan plane, and the "region of validity" is defined by the angle between the edge of the AUT and the edge of the scan plane. In some applications, results are required over a larger angular region than is possible with the available scanner. The angular coverage can be increased by rotating the antenna and repeating the measurement. The results of the two measurements are then combined. Successful combination depends on using both the coordinate system and vector components that are appropriate for the antenna rotation.

In general for a single antenna orientation, any coordinate system such as (x, y, z); (k_x, k_y, k_z), (θ, ϕ); (α, ε); or (A, E); can be used with any vector components since the coordinate

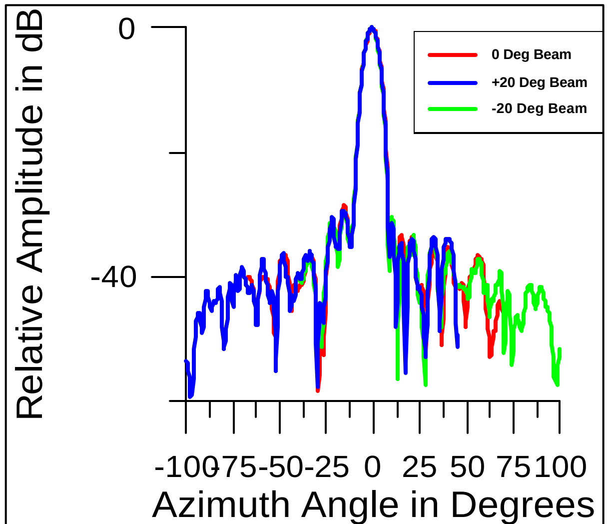

Figure 7 Illustrating the increase in angular coverage by combining steered beams.

system defines a direction or location in space and the vector components define two orthogonal components of the electric field. In a sense the "natural" coordinate system is the one that corresponds to the vector components since the field vectors are along the lines of the corresponding coordinates. In the specific case of a rotated coordinate system, there is a compelling reason to use a specific coordinate system and a specific set of vector components as the "initial" or "natural" set. For rotation about the z-axis, θ, ϕ coordinates and vector components are the natural choice. For rotation about the x-axis, α, ε coordinates and vector components are the natural choice. For rotation about the y-axis, A, E coordinates and vector components are the natural choice. If these natural coordinates and components are chosen there will not be any change in the shape of the pattern due to rotation, only an offset. Once the offset has been taken care of, we can always convert to any other components or coordinates.

Figure 6 shows the antenna that was used for the following tests in its initial orientation with the main beam normal to the scan plane. It is an X-Band slotted array approximately 14 inches in diameter. With a measurement length of 60 inches in x and y, and a z-distance of 8 inches, the resulting far-field pattern is valid over ±70 degrees in both azimuth and elevation. To increase the angular coverage in the azimuth direction, the antenna was rotated +20 degrees about the y-axis using a rotator that was aligned with its axis of rotation parallel to the y-axis of the scanner. This corresponds to the Azimuth angle of the coordinate system shown in Figure 3. Near-field measurements were repeated, for this antenna orientation and for a similar rotation of –20 degrees in Azimuth. Figure 7 shows the results of these three measurements along the horizontal principal plane. The +20 degree and –20 degree patterns have been shifted to put the peak on-axis for this display. The azimuthal coverage has now been increased to ± 90 degrees and it is apparent that the patterns show very good agreement within their common regions of validity.

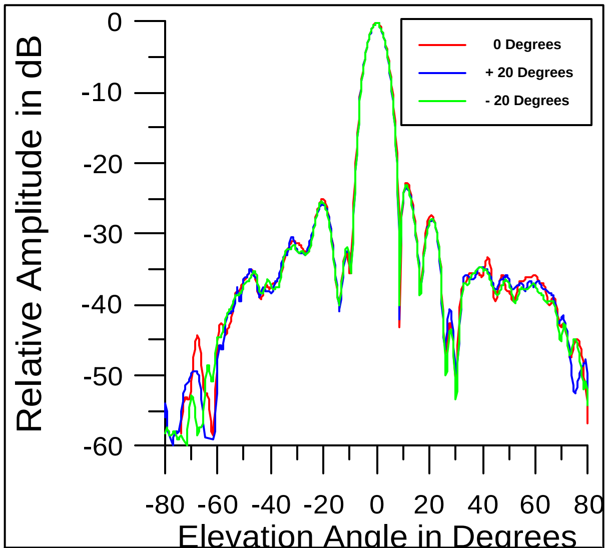

Figure 8 V-cut through beam peak for three Azimuth rotations of AUT. AZ EL coordinates and vector components used.

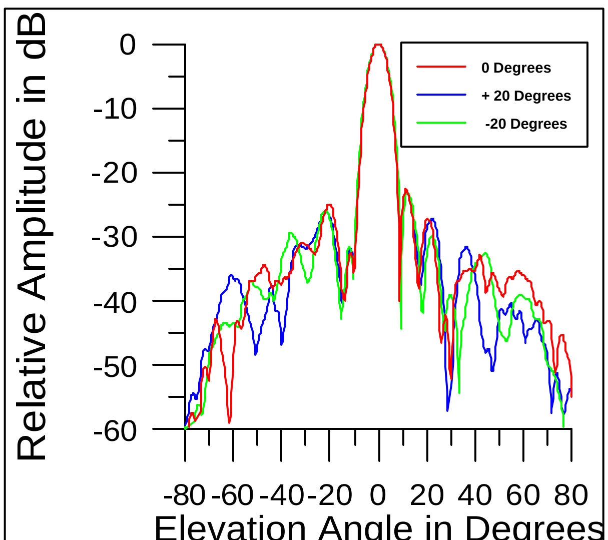

Figure 9 V-cuts through the beam peak for three Azimuth rotations of the AUT. Alpha Epsilon coordinates and vector components.

These patterns have used A E coordinates and vector components that are the natural choice for rotation about the y-axis. With this choice, the patterns should be shifted in Azimuth and the shape of the patterns should not change. This is illustrated in Figure 8 where vertical cuts through the peaks of the beams are plotted for the 0 degree, -20 degree and +20 degree antenna orientations. These patterns show excellent agreement, that is in contrast to Figure 9 which is a similar comparison of the V-cuts using α-ε coordinates and components. The comparison here is especially poor at angles far off axis.

The differences in the patterns in Figure 9 do not mean that there is an error in the measurement or that it is "incorrect" to use the α-ε coordinates and components. All three curves are a correct and valid representation of the field for the given situation. The patterns are different because the shape of the pattern changes when using these coordinates and components with a rotation about the y-axis. If the shifted patterns are going to be combined, or if the shifted patterns are going to be compared and the differences used to estimate errors in the measurement, A E coordinates and vector components must be used with a rotation about the y-axis.

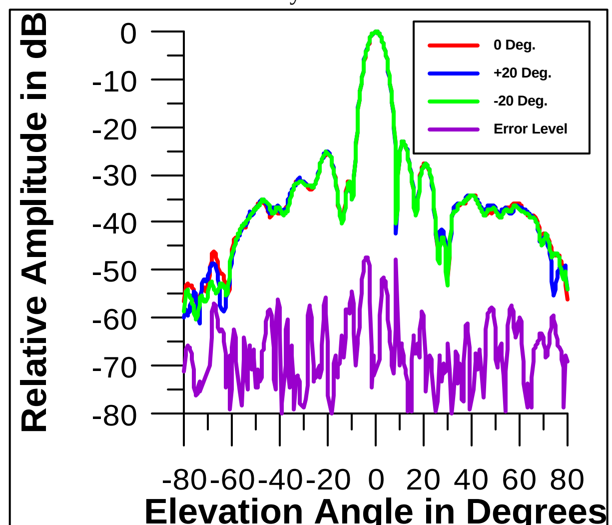

Figure 10 Comparison of V-Cuts through beam peak for three rotations of AUT after averaging 4 results for each beam rotation.

7. ESTIMATING ERRORS IN PLANAR NEAR-FIELD MEASUREMENTS

Comparing the results of two different near-field measurements can be an effective tool for estimating errors if the measurement system is changed in a known way between measurements[2]. The change in the measurement system is designed to identify one or more sources of error while leaving other sources unchanged. Rotation of the antenna about one axis can identify some sources of error that are difficult to quantify with other tests as illustrated by the following examples.

If the correct coordinates and components are used for a given rotation, if the rotation is precisely known, and if there are no errors in the measurement system, the original and rotated patterns should agree exactly. For the azimuth rotations previously described, the rotations were carefully controlled and known, and the differences seen in Figure 8 are due to measurement errors. Multiple reflections between the AUT and the probe produce some errors in every near-field measurement. This error can be identified and partially corrected by taking measurements at a series of 4 z-distances in increments of λ/8. This was done for all three of the azimuth beam rotations and the four far-field measurement results for each beam rotation were averaged to reduce the effect of the multiple reflections. The average far-fields for the three beam rotations were then compared as shown in Figure 10 and a residual error signal calculated that would cause the differences in the patterns. In the main beam region, where the multiple reflection error is the largest, the residual error level is approximately –50 dB while in the sidelobe region it has been reduced to approximately –60 to –65 dB. The error sources that should be identified by the steering of the beam in Azimuth are X-position errors, phase linearity and room reflections in the Azimuth directions. From the results shown in Figure 10, the combined effect of these error sources is less than -60 dB below the main component peak.

The beam rotation tests were repeated for Elevation rotations

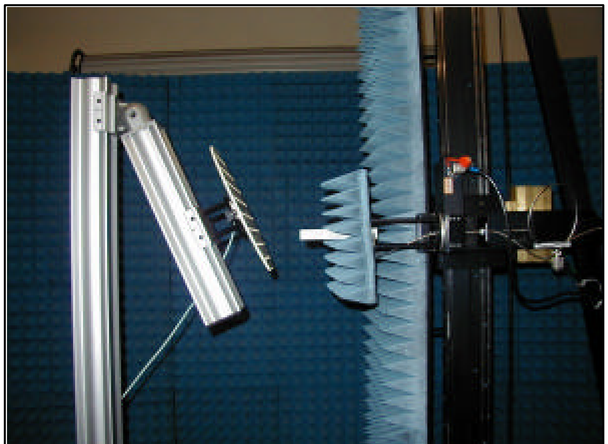

Figure 11 Antenna rotated in Elevation 24.5 degrees.

of 8.5 and 24.5 degrees. It was not possible with available equipment to control the rotations as precisely for these measurements, and so the comparisons were not as good as for the Azimuth rotations. The results do illustrate the identification of a different source of error as shown in Figure 12. The large error signal of approximately -35 dB for Elevation angles between –30 and –70 degrees is due to scattering from either the metal support behind the antenna or the floor. The effect of this scattering is evident in the near-field data where the amplitude shows local peaks at the bottom of the scans as indicated by Figure 13. With this information, additional absorber could be added behind the antenna or on the floor to further reduce the effect of this scattering.

8. CONCLUSIONS

Four different spherical coordinate systems and the associated vector components associated with these coordinates have been defined. Far-field or Near-field measurements can be presented using any of these systems, and each is a valid representation of the antenna pattern and polarization. If comparisons are made between different measurements or between measurements and theoretical calculations, the coordinates and components must agree. Also if the antenna is rotated about one axis and measurements are combined, the proper coordinates and components must be used that will not change the shape of the pattern for the rotation.

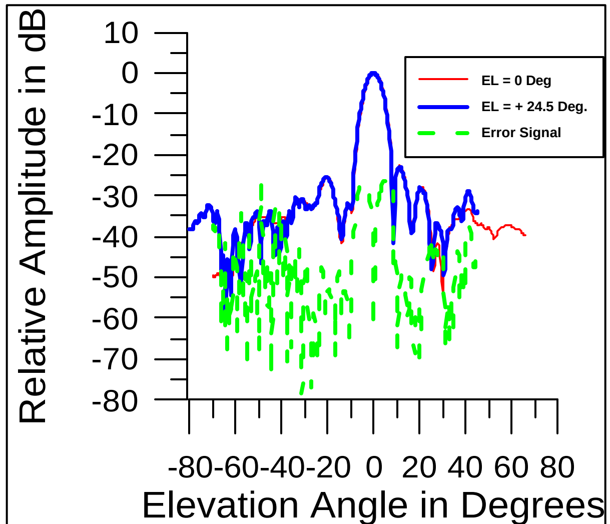

Figure 12 Results of comparing Averaged patterns for 0 degrees and +24.5 degrees Elevation beam rotations.

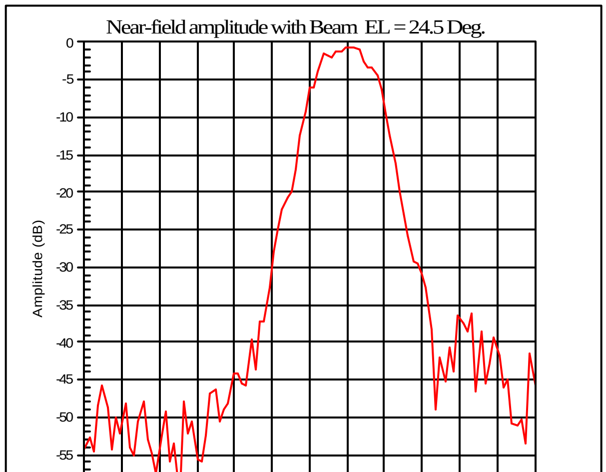

Figure 13 Y-cut near-field amplitude for beam steered 24.5 degrees in Elevation.

9. REFERENCES

[1] A. C. Ludwig, "*The definition of cross polarization, IEEE Trans. Antennas Propag. Vol 21, No. 1 pp 116-119, January 1973.*

[2] A. C. Newell, "*Error Analysis Techniques for Planar Near-Field Measurements", IEEE Trans. Antennas Propag., Vol 36, No. 6, pp. 754-768, June 1988.*