FAST AND ACCURATE ANTENNA ALIGNMENT CORRECTION PERFORMED USING A VECTOR ISOMETRIC ROTATION

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Abstract: The success of most traditional implementations of antenna measurement techniques whether near field, far field or compact, assume that a fiducial mechanical datum associated with the antenna under test (AUT) can be accurately and precisely aligned to the mechanical axes of the test range. Unfortunately, an alternative approach is sometimes necessary, as achieving such careful alignment is not always convenient or possible [1]. Instead, if the relationship between the frame of reference associated with the antenna and the frame of reference associate with the range can be acquired, *i.e.* assuming that it is known, then in principal any misalignment can be corrected for within the data processing chain [2]. Techniques for rigorously implementing the necessary vector isometric rotation are well documented and usually utilise the concepts of a modal expansion [3, 4]. In general this is not always convenient as these methods can be difficult to implement and often require the transformation of one modal expansion to another, *e.g.* planar or cylindrical to spherical, *etc.*. This paper describes the additional post processing that is required to yield alignment corrected far field data from an acquisition of an imperfectly aligned antenna. A general-purpose vector isometric rotation strategy is utilised that is reliant upon interpolation, rather than a particular modal expansion. The interpolation is performed using a polar, *i.e.* amplitude and phase, implementation of a two dimensional bi-cubic convolution interpolation algorithm [5]. The effectiveness of this technique is then demonstrated through the use of range measurements.

Introduction: The normal purpose of a range measurement process is to characterise the radiation pattern of the AUT at a very great, or infinite, distance with reference to an angular, or other co-ordinate set, with respect to a carefully defined mechanical interface. Once acquired, this data can then be utilised to establish the extent to which the instrument fulfils its requirements which is often accomplished by comparing the measured performance with a theoretical prediction. In principal it is possible for the AUT to be orientated in such a way as to be aligned perfectly to the principal axis of the range. However, in practice, either due to the size of the antenna or a limitation of the mechanical mounting structure, there are often considerable difficulties inherent with positioning the assembly with the degree of accuracy and precision required. The use of rigorous alignment correction methods can enable very accurate antenna to range alignment to be achieved in cases where alternative strategies would perhaps fail or require the use of additional mechanical alignment and support equipment, that would potentially need to be large, cumbersome, expensive, and which would likely be highly project specific.

The acquisition of the crucial antenna-to-range alignment data will vary from facility to facility. Within a spherical system an optical alignment method is often employed [2]. Conversely, a planar facility may utilise a procedure that involves touching-off contact points on the antenna using a precision mechanical contacting probe [2, 3]. Such alignment information can be written down and stored in several different but equivalent ways, *e.g.* with: an ordered series of passive rotations (*e.g.* triad of Euler angles), a direction cosine matrix, *etc.*. Whichever method is chosen the application of this data to the correction of measured antenna radiation patterns will be by means of a vector (*i.e.* pattern and polarisation) isometric rotation. Generally, the fields are expanded onto a modal basis that is commensurate with the geometry of the measurement system. The mode coefficients are then rotated rigorously, *i.e.* avoiding recourse to approximation, whereupon the fields can be reconstructed in the rotated co-ordinate system. For example, for the case of a planar range, the application of alignment correction data is applied rigorously by expanding the plane wave spectrum on an irregular grid (*i.e.* non-rectilinear) in the range system. This irregular space corresponds to a regular angular domain in the antenna mechanical system. The field components then need transforming from the range polarisation basis into the antenna polarisation basis. However, a similar result can be obtained with only a small increase in uncertainty that yields a significant reduction in the amount of computational effort, and therefore time, required by recourse to approximation. Often approximation, in the form of piecewise polynomial interpolation, is avoided due to fears concerning the accuracy attained. However, when deployed correctly, and used in suitable applications, the power and speed of this approach can be of utility.

Method: Transformation matrices are matrices that post-multiply a column point vector to produce a new column point vector. A series of transformation matrices may be concatenated into a single matrix by matrix multiplication. A transformation matrix may represent each of the operations of translation, scaling, and rotation. However, if A is a three by three orthogonal, normalised, square matrix, it may be used to specify an isometric rotation that can be used to relate two frames of reference, *i.e.* two co-ordinate systems. Here, an isometric rotation is taken to mean a transformation in which the distance between any two points on an object remains invariant under the transformation. Any number of angular definitions for describing the relationship between the two co-ordinate systems are available. However, if the angles azimuth, elevation and roll are used, where the rotations are applied in this order, we may write that a point in one frame of reference can be specified in terms of a point in the other frame of reference as,

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [A]\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(roll) & \sin(roll) & 0 \\ -\sin(roll) & \cos(roll) & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(el) & -\sin(el) \\ 0 & \sin(el) & \cos(el) \end{bmatrix}\cdot\begin{bmatrix} \cos(az) & 0 & -\sin(az) \\ 0 & 1 & 0 \\ \sin(az) & 0 & \cos(az) \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{1}$$

Here primed co-ordinates are used to denote the rotated co-ordinate system. These transformation matrices can be easily derived either from geometry, or from trigonometric identities. Here, in accordance with the rules of linear algebra, the first rotation matrix is written to the right with the next rotation being written to it's left and so on. Such rotations are termed passive as each successive rotation is applied to the newly rotated system. Specifically then, the rotations are understood to have been performed in the following order: 1) rotate about the negative y-axis through an angle azimuth, 2) rotate about the negative x-axis through an angle elevation, 3) rotate about the z-axis through an angle roll. Thus, when az = el = roll = 0 the direction cosine matrix will be the identity matrix and specifies that the primed and un-primed systems are coincident and synonymous. When direction cosine matrices are used in this way, the determinant of the matrix should be calculated as any deviation from unity can be treated as an indication of error. Furthermore, a determinant of –1 indicates that a right-handed co-ordinate system has been transformed into a left-handed one and vice-versa. For the case of applying a scalar isometric rotation to a far field antenna pattern, *i.e.* one in which the reference polarisation is rotated with the pattern, this can be accomplished readily by converting the rectangular plotting x- and y- axes into the equivalent direction cosines before using equation (1) above to compute the rotated direction cosines. Once the rotated direction cosines are known they can be used to compute the new rectangular plotting x- and y- axes in the rotated system. This process can be used to rotate an antenna pattern that has been tabulated in any of the conventional tabulating, *i.e.* plotting, co-ordinate systems, *e.g.* azimuth over elevation, elevation over azimuth, polar spherical, true-view, direction cosine, *etc.*. Once the new desired co-ordinates are known it is a simple matter of using a convenient interpolating scheme to approximate the value of the underlying pattern at each of the new co-ordinates. By utilising Cartesian direction cosine components and by wrapping any resulting spherical angles into their equivalent ±180° range, this procedure is capable of correctly handling arbitrary rotations to complete full sphere patterns.

To this end, the antenna pattern data must be interpolated from the sampling grid onto the desired output grid. However, for the case of a band-limited function this interpolation need not introduce any approximation as the entire function can be reconstructed using the sampling theory. And in essence, this exactly how conventional active alignment correction techniques operate. Thus although rigorous, such an approach is computationally intensive and often the uncertainties introduced by other interpolation schemes are found to be acceptably small. When the nature of the underlying function is not to rapidly varying when compared to the sampling interval, fitting a polynomial function to the sampled pattern in a piecewise sense can approximate the underlying function from samples taken on a regular, equally spaced two-dimensional grid. Many techniques are available for this purpose, however bi-cubic convolution [5] has been found to be both efficient and robust, supplying data that is smooth, *i.e.* both the function and its derivative are continuous. Here however, the interpolation was performed in terms of the polar, amplitude and phase, form of voltage rather than the equivalent rectangular, real and imaginary, form. This is often preferable as more reliable results are obtained in the presence of a phase taper across the sampled function. This follows from the fact that in polar form, even a large phase taper only equates to a constant amplitude function and a linear phase function, whilst in rectangular form this equates to a highly oscillatory real and imaginary function that is difficult to approximate reliably with piece-wise polynomial functions. When interpolating phase data care must be taken to wrap all local phase points into the same phase range before estimating the value of the interleaving point. In practice, the unwrapping of two dimensional phase data is a demanding task. Consequently, the task is significantly simplified here by unwrapping the phases of only the nearest sixteen points (*i.e.* only those points required by the bi-cubic convolution interpolation algorithm) to the location of the desired interleaving point. The algorithm can be expressed as: 1) locate the sixteen nearest neighbour points, 2) unwrap all points into the same phase range as the nearest neighbour point, 3) apply conventional interpolation algorithm, 4) wrap the interpolated value back into the conventional ±180° range. If the rate of change in the phase function is large, *i.e.* greater than ±180° between sampling points, this phase interpolation scheme will become unreliable. However, this not generally a problem as this would also invalidate the sampling theorem.

As described above, the by-product of a scalar rotation is that the reference polarisation is rotated with the pattern by exactly the same amount as the pattern has been rotated. Thus, an inverse rotation must be applied to the polarisation basis in order that it can be returned to its original position. The algorithm for applying a vector rotation to an antenna pattern can thus be expressed as follows: 1) Resolve the far field pattern (*e.g.* resolved onto a Ludwig III polarisation basis) onto a Cartesian polarisation basis. 2) Calculate the equivalent direction cosines from the plotting co-ordinates, *e.g.* calculate α, β and γ from the azimuth and elevation angles, and apply an isometric rotation. Calculate the rotated plotting co-ordinates from the rotated direction cosines. 3) Interpolate each of the Cartesian field components from the tabulated plotting grid onto the rotated plotting grid using a preferred interpolating scheme. 4) Apply an inverse isometric rotation to the field components to complete the vector isometric rotation. 5) Resolve the far field pattern back onto the desired polarisation basis (*e.g.* Ludwig III definition).

Results: In order that the validity of the alignment techniques could be tested, a low gain antenna was acquired at a variety of different orientations with respect to the axes of the range. In each case the antenna to range alignment was acquired using a precision mechanical contacting probe and the data transformed to the far field. The corrected far field data can be seen below presented in terms of iso-level (contour) plots for the case where the AUT was misaligned grossly in azimuth, elevation and roll. The antenna pattern has been tabulated using a an azimuth over elevation co-ordinate system and the electric field was resolved onto a nadir-centred Ludwig III polarisation basis.

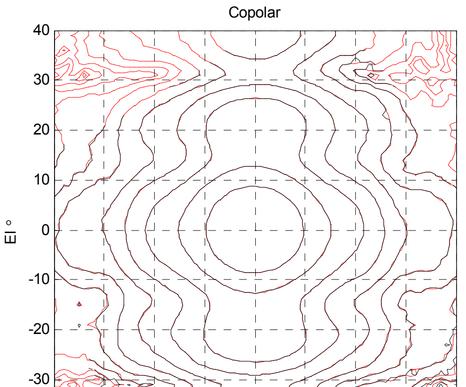

Figure 1: Far field Copolar Pattern.

Here, red contours denote results obtained using conventional active alignment correction whereas, black contours denote results obtained using vector isometric rotation and polynomial interpolation and the contours have been plotted at 5dB steps from 0dB to –70dB. The data has only been plotted out to ±40° in azimuth and elevation in order that the entire far field data set should be free from first order truncation effects. The agreement attained between the respective results is encouraging as the differences observed are small. The verification of the conventional active alignment correction technique is not the purpose of this paper and is a subject in its own right. However, a detailed description of that method and its verification can be found in [1, 2, 3]. The visibility of red contours at the edges of the plot is merely an artefact of the uncorrected data being known only out to ±40° in azimuth and elevation and is not a fundamental limitation of the procedure. To further verify that the alignment information had been applied correctly, the nominally aligned data set was rotated through 10° in azimuth, then 20° in elevation and finally through 30° in roll. The resulting rotated data set was then further transformed by the inverse rotation, *i.e.* $[A]^{-1}$, and compared against the original nominally aligned pattern. This comparison can be seen presented in figure 2 below.

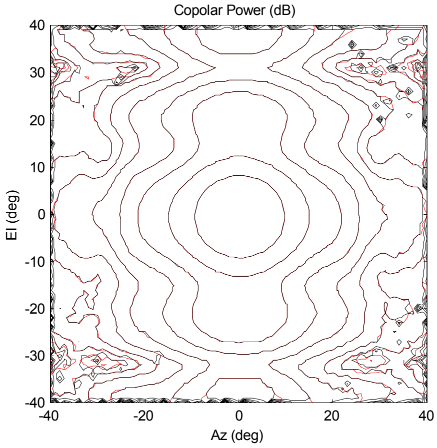

Figure 2: Verification of vector rotation algorithm copolar pattern.

Here, red contours represent the original data set whilst the black contours represent the twice-rotated data set and the contours have been plotted at 5dB steps from 0dB to –70dB. Clearly, the agreement is encouraging with only minor differences resulting in inaccuracies introduced by approximations inherent within the interpolation algorithm. The errors observed at the edges of the recovered data set result from a failure of the interpolation algorithm to correctly handle edge elements. Clearly, approximate methods such as these can prove unreliable on occasion. For example, the results may become unreliable if used to correct very high gain antenna patterns where the underlying pattern function fluctuates rapidly with respect to the sampling interval. Errors may also be encountered in regions of low signal levels where the presence of noise can disturb the interpolation, *e.g.* in pattern nulls. The interpolation formula use several neighbouring points to approximate an interleaving point. Thus, although being suppressed localised noise will be spread out to any interpolated point using a noise contaminated sampling node. However, for many cases where the rotation has been accomplished using approximation, rather than rigorous sampling theorem (*i.e.* Whittaker) interpolation, the results are usually sufficiently reliable for the purposes of visualisation.

Conclusion: This paper has shown that a simple vector isometric rotation, employing an efficient polar implementation of a bi-cubic convolution interpolation algorithm, can be used to obtain reliable alignment corrected results that are in very close agreement with those results obtained by more intensive, rigorous means. Although it is always possible to choose a function that is sufficiently pathological to make a mockery of any interpolating procedure, in practice this implementation has been found to be comparatively robust. Finally, the utilisation of such realignment processes substantially increases the ability of a given measurement facility to accurately characterise antenna assemblies, even in circumstances where gross misalignment is unavoidable, or when high degrees of angular precision are required.

References

[1] S.F. Gregson, J. McCormick, "The Application Of Non-Rectilinear Co-Ordinate Systems In The Characterisation Of Mis-Aligned Space Antennas", AMTA 1999.

[2] S.F. Gregson, A.J. Robinson, "An Inter-range Comparison in Support of the Characterisation of Space Antenna Systems and Payload Testing", IEE, 1988.

[3] S.F. Gregson, J. McCormick, "Image Classification as Applied to the Holographic Analysis of Mis-Aligned Antennas", ESA ESTEC, 1999.

[4] A. Frandsen, "Spherical near-field transformation program with probe correction", TICRA, Denmark, 1995.

[5] R.G. Keys, "Cubic Convolution Interpolation for Digital Image Processing", IEEE Trans. on Acoustics, Speech, and Signal Processing, Vol. 29, No. 6, Dec. 1981, pp. 1153-1160.

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