# PROPOSAL FOR A NOVEL NEAR-FIELD ANTENNA MEASUREMENT TECHNIQUE EMPLOYING A CONIC FRUSTUM GEOMETRY 

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#### Abstract

A technique for the prediction of far-field antenna patterns from data obtained from modified plane-polar nearfield measurement system is proposed. This technique utilises a simple change in facility alignment to enable near-field data to be taken over the surface of a conceptual right conic frustum. In this way, existing planar facilities can be modified so that they are able to characterise the wide angle antenna performance in situations where hitherto they would have been limited by truncation. Furthermore, this system offers a cost effective solution to the characterisation of a class of antenna that currently can only be effectively served with spherical near-field scanning. This paper presents the measurement technique, proposes the probe corrected transformation algorithm and presents preliminary results before outlining the future work.


## 1. Introduction

It is well known that far field antenna parameters such as pattern, gain, directivity, beamwidth, etc., can be derived analytically from near field measurements. For such parameters, which are not obtained directly from measurements made in the near-field, a transformation from one spatial domain, or surface, to another is necessitated. This transformation, of monochromatic but otherwise arbitrary waves, can be accomplished by representing the field at an arbitrary point in space as an integral over the surface on which the fields are known (1). Alternatively, considerable computational advantages can be obtained by representing the field as a summation of any elementary wave solutions to Maxwell's equations (2). Here, the coefficients to these solutions are determined by matching the fields on the surface on which the fields are known and by using mode orthogonality. Solving this modal expansion for the fields at an infinite distance from the radiator results in the far field pattern.

In the latter case, a degree of mathematical convenience can be obtained from selecting a modal basis that is commensurate with the measurement geometry, i.e. by utilising plane waves, cylindrical waves, or spherical waves respectively for the case where the measurements are taken over planar, cylindrical or spherical surfaces. The principal factor in determining the complexity of 1 ; the near field to far field transformation 2 ; the probe pattern correction and 3; the extent of the truncation of the data, is the geometry of the surface over which the near field is sampled, and its associated orthogonal modal basis. However, the utility of any such measurement technique also depends upon the ease with which the robotic positioning apparatus can be constructed. The aforementioned geometries have the inherent advantage that they can be readily realised from rotation, and linear translation stages at an economically attractive price. However, the Helmholtz differential equation can be solved by separation of variables in a number of other co-ordinate systems, eleven in total in three-dimensional Euclidean space, and some of these hitherto unused systems can also used as a basis for the construction of a near-field antenna measurement technique. Possibly the most obvious candidate for this treatment is the right conical co-ordinate system $(3,4)$. This system constitutes a particularly attractive proposition as the robotic positioning system, can be conveniently fabricated using readily available, commercial off the shelf (COTS) positioning stages.

## 2. Overview of the Frustum Measurement Technique

Conceptually, the right conical measurement system is perhaps most closely related to the well documented, well understood plane-polar near-field scanning technique. Only here, the axis of rotation of the antenna under test (AUT) and the linear translation stage which caries the probe, are no longer constrained to be exactly antiparallel with one another. By taking samples incrementally on a raster grid by varying the polar angle $\phi$ and radial displacement $r$ the near electric-field can be sampled over the surface of a right code. The conical measurement is illustrated schematically in Figure 1 below.


Figure 1: Conical measurement co-ordinate system.
Here, the AUT is shown aligned so that its bore-sight is orientated along the negative $z$-axis. If the near-field probe is rotated through $90^{\circ}$ about its axis and this process is repeated, two orthogonal tangential electric field components can be acquired. This in principle is sufficient to enable the entire electromagnetic six-vector to be determined throughout that portion of, free, space outside of the conceptual measurement cone that encloses the AUT. The principal advantage of the plane-polar, and thus conical, scanning technique is that the mechanical complexity of the robotic positioning system is comparatively small as only a single linear probe trajectory is required. In principle then, any existing plane-polar facility could be converted into a right conical near-field antenna measurement system merely by altering the alignment of the system. Crucially, this would potentially enable any modified system to obtain far-field antenna pattern data out to polar angles of greater than $90^{\circ}$, i.e. over more than a half-space, with very little additional increase in complexity of either mechanical or control sub-systems. This is a feat that is beyond any conventional planar system. In principle the z -axis of the range could be orientated so that it is aligned with the local gravity vector, making this an attractive configuration for the characterisation of gravitationally sensitive antenna assemblies. It is often the preferable when taking nearfield antenna measurements that a measurement geometry be selected which is commensurate with the geometry of the AUT. Thus, this technique would be particularly well suited to the characterisation of base-station antennas, or arrays installed behind tangent ogive radomes, such as those commonly employed with nosemounted fire-control radars. This is an electrically large system that often presents the experimentalist with both electromagnetic and mechanical challenges.

Unfortunately, any imperfection in the alignment of the conical system could result in the introduction of significant errors in the corresponding far-field pattern. This is a consequence of the fact that naturally, the boresight direction of the AUT, and thus the region of greatest field intensity will be directed towards the tip of the cone, which is where the set of radial conical linear cuts intersect and where the alignment issues are most critical. Obviously, this can be eased by orientating the AUT so that it "looks" out through the side of the cone thus avoiding the tip region, but this is perhaps an inelegant solution. One alternative that has been used with considerable success in the closely related poly-planar measurement technique is to us a flat-topped measurement surface (3). For the poly-planar case a truncated pyramid, i.e. a pyramidal frustum, was employed to resolve this difficulty. Here however, an analogous conical frustum would be used which is a frustum created by slicing the top off a right cone where the cut is made parallel to the base of the cone. This arrangement is illustrated together with the conventional conical arrangement in Figure 2 and Figure 3 below. Here, the cap that is used to replace the tip of the conic section constitutes a conventional plane polar measurement. It is intended to displace the intersection between the individual cuts from the region of greatest field intensity to a less sensitive location. Thus, in the event that the adjacent scans do not intersect perfectly, the resulting positional error will impact less on the far field pattern.


Figure 2: Conical frustum measurement co-ordinate system.


Figure 3: Conical measurement co-ordinate system.

## 3. Overview of Proposed Frustum Transform and Preliminary Results

Unfortunately, and as is the case for the poly-planar technique, the departure from a perfect primitive geometry necessitates that a hybrid transform be utilised. Generally, modal expansion techniques are inappropriate for use with bespoke sampling surfaces, as the sampling surface must correspond to a constant co-ordinate surface in the system for which the harmonic function series solutions are available. Essentially, the proposed frustum nearfield to far-field transform can be considered to be an extension, or limiting case, of the hybrid plane-wave spectrum physical optics poly-planar transform. Although in this implementation, it is more convenient to employ a conventional plane polar transform to process the flat disk that is produced by cutting off the top of the right cone. If however, the sampled data set is truncated, the reconstructed normal field component will be in error as a result of Gibb's ripple. This is obviously a problem in this case as it is very likely that the edges of the disk will be illuminated by significant field intensities as it is essentially a partial scan which are by definition, truncated. Furthermore, these probe corrected field components are required before the partial data sets can be combined, as the principal of superposition requires that each component be resolved onto the same polarisation basis. Fortunately it has been found that such difficulties can be resolved with the use of a simple windowing technique (3). This involves applying a known window to the measured data prior to processing to force adherence to the Dirichilet boundary condition. Once the probe-corrected electric and magnetic fields are recovered, the windowing function is divided out of the reconstructed quantities to obtain reliable results. Although here this has to be implemented for a circular domain, whereas previously it was used with a rectangular domain this only a minor change. Providing this procedure can be used to handle linear strips then the Kirchhoff-Huygens $(\mathrm{KH})$ formula can again be used to transform the reconstructed probe-corrected electromagnetic field to the far-field (1). This method essentially constitutes a direct integration of Maxwell's equations with the use of a vector Green's theorem to yield an integral solution of Maxwell's equations in terms of sources, c.f. the Stratton-Chu solution. It is applicable to arbitrary shaped apertures over which both the electric and magnetic fields are prescribed. The far electric field, at a point $P$ radiated by a closed Huygens surface $S$ is,

$$
\begin{equation*}
\underline{E}_{p}(\underline{\hat{u}})=\frac{\pi}{j k_{0} \lambda^{2}} \int_{S}\left[\underline{\hat{u}} \times(\underline{\hat{n}} \times \underline{E})+Z_{0} \underline{\hat{u}} \times\{(\underline{\hat{n}} \times \underline{H}) \times \underline{\hat{u}}\}\right] e^{j k_{0} \underline{\hat{u}} \underline{\underline{r}}} d a \tag{1}
\end{equation*}
$$

This expression yields the far-field vector pattern function from an integral of the electric and magnetic fields over the closed surface $S$ and $d a$ is an elemental area of $S$ and $\underline{n}$ is the outward pointing surface unit normal. Here, the $r^{-1}$ term and the unimportant phase factor have been suppressed as in accordance with usual convention. This is exactly the same algorithm that has been used successfully to calculate the far-field of installed antenna assemblies when the enclosing radome is a tangent ogive (5).

Thus, the success of this technique depends greatly upon the validity of the partial-scan windowing technique to recover the entire probe-corrected electromagnetic six vector in the limiting case where the domain collapses to a one-dimensional line. Clearly, for the windowing technique to be deployable the partial scan must be sufficiently large to enable the application of a smooth amplitude taper. However, as we are only interested in the results at a single line in space, if the near field data were to be extrapolated such that a windowing function could be usefully applied then, although the extrapolated results would be in error, the central point would be handled correctly. The process would consist of the following steps: extrapolate a sufficiently large data set, apply the near field window, reconstruct the normal field component, remove the window, and remove the extrapolated points so that only the normal field component corresponding to the original data points remains.

As in the limit only a single field point is known, the only form of extrapolation available is by means of a constant, i.e. set the extrapolated data equal to the known field point and allow the windowing function to taper this smoothly to zero. This can be seen presented below in Figure 4. Here, a cosine windowing function has been utilised but similarly encouraging results have been obtained from other windowing functions, e.g. triangular.


Figure 4: Synthesized $x$-polarised near-field pattern.


Figure 5: Comparison of reconstructed normal field component recovered from full two-dimensional pattern and from a single cut.

Figure 5, shown above, contains a cut that compares the normal field components having been reconstructed using the entire two-dimensional data set, and the equivalent having been recovered using the extrapolation/windowing method on a single near field cut. From inspection of Figure 5 it can be seen that the degree of agreement is very encouraging with the respective traces being in good agreement over the entire near field region and similarly good results have been attained for the phase pattern. Although further verification is ongoing, this would suggest that this technique could perhaps be used to obtain probe-corrected far field data from measurements taken using any scanning geometry that can be decomposed into a series of one-dimensional linear cuts, i.e. cylindrical and relevant here, conical measurement systems.

## 4. Summary and Conclusion

In this paper, a novel near-field antenna measurement technique has been proposed. Although this technique has been based upon the existing plane-polar near-field methodology which enables existing readily available hardware to be utilised, it has been modified to offer significant benefits including the ability to characterise wide out antenna performance. This, makes it particularly well suited to the characterisation of base station antennas and tangent ogive radome characterisation. A novel probe-corrected near-field to far-field transform algorithm has been proposed and certain critical components have been verified by means of measurement and numerical simulation. Finally, this paper recounts the progress of an ongoing research study. Consequently, several issues remain to be addressed and the future work is to include obtaining verification of the success of the right conic frustum measurement technique through further numerical simulation and experimentation.

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