Theory of near field Measurements:

The theory of near field measurements for the characterisation of antenna has been discussed and reviewed in several papers, G. T. Whittaker, G. N. Watson, “Modern analysis”, J. A. Stratton, “Electromagnetic theory”, H. G. Booker, P. C. Clemmow, “The concept of an angular spectrum of plane waves, and its relation to that of polar diagram and aperture distribution”. Some of those discussions are presented below.

In a source free space region, which is homogeneous, linear and isotropic medium free of sources, harmonic time variations and assuming the conductivity of the medium is zero, Maxwell equations can be transformed into the following vector wave equations,

$$\nabla^2 \underline{E} + k^2 \underline{E} = 0$$
$$\nabla^2 \underline{H} + k^2 \underline{H} = 0$$
$$\nabla \cdot \underline{E} = 0$$
$$\nabla \cdot \underline{H} = 0$$

Where,

$\underline{E}$ is the electric field

$\underline{H}$ is the magnetic field

The general solution for $\underline{E}$ can be constructed from a linear combination of,

$$\underline{E} = \underline{A}(k) e^{-j\underline{k}\cdot\underline{r}}$$

Where,

$\underline{A}(k)$ is the vector amplitude of the wave, also known as the plane wave spectrum (PWS) of the field

r is the position vector, i.e. the distance co-ordinate of a spherical co-ordinates system centred at x=y=z=0

k is the vector wave number.

and,

$$\underline{k} = k_x \underline{x} + k_y \underline{y} + k_z \underline{z}$$
$$\underline{r} = x\underline{x} + y\underline{y} + z\underline{z}$$

So the dot product of k yields the square of the scalar quantity as follows.

$$\underline{k} \cdot \underline{k} = kx^2 + ky^2 + kz^2 = k^2 = \omega^2 me$$

Substitution of the solution of E into the square of the scalar quantity yields,

$$\underline{k} \cdot \underline{k} = k^2 = \omega^2 me$$

Therefore, if z is assumed to be the direction of propagation at a fixed frequency only two components of k, namely k_x and k_y can be independently specified for such a case. The imaginary kz refers to the evanescent mode which dies down after approximately a wavelength in the direction of propagation from the aperture plane. Substitution of the solution of E in the divergence equation yields,

$$k_x A_x(\underline{k}) + k_y A_y(\underline{k}) + k_z A_z(\underline{k}) = 0$$

For the assumed direction of propagation, the plane wave spectrum for x and y dimensions are thus,

$$A_x(\underline{k}) = -\frac{1}{k_x}\left(k_y A_y(\underline{k}) + k_z A_z(\underline{k})\right)$$

and

$$A_y(\underline{k}) = -\frac{1}{k_y}\left(k_x A_x(\underline{k}) + k_z A_z(\underline{k})\right)$$

Now,

$$A\left(k_x, k_y\right)e^{-j\underline{k}\cdot\underline{r}}$$

This represents a plane wave propagating in the k direction. It can be shown that the following expressions constitute a solution for the above equations. For z ≥ 0 and satisfied the prescribed boundary conditions on the plane z=0, namely: The wave propagates tangential to the aperture plane, the field is zero outside the aperture and the field is not pathological within the aperture itself

$$\underline{E}(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A\left(k_x, k_y\right)e^{-j\underline{k}\cdot\underline{r}}dk_x dk_y$$

$$\underline{H}(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{k \times} A\left(k_x, k_y\right)e^{-j\underline{k}\cdot\underline{r}}dk_x dk_y$$

A is said to be a plane wave spectrum because it is considered to be a summation of plane waves. For planar near field measurements the antenna is placed in the region z ≤ 0 as shown in figure 3 below.

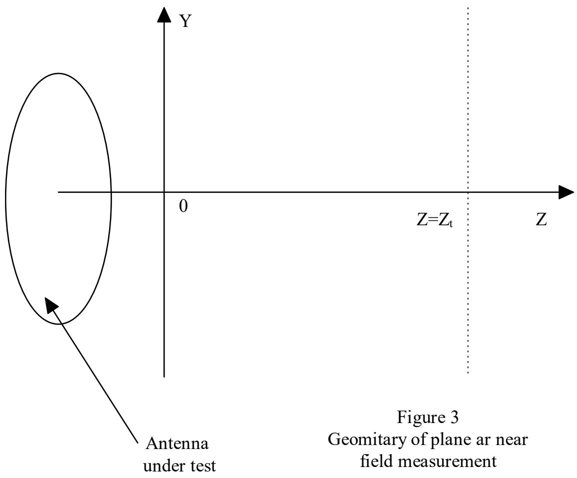

Figure A1

The probe must be both in the near field and outside the reactive field for the above theory to apply. Planar scanning is conducted on a plane, specified by z=z$_t$, near the AUT. For this simulation we have assumed an ideal probe which requires no phase correction. This assumption is made to reduce the problem to it's most fundamental elements.

$$k_z = \begin{cases} \left(k^2 - k_x^2 - k_y^2\right)^{\frac{1}{2}} & \text{if } k_x^2 + k_y^2 \le \mathrm{k}^2 \\ -j\left(k_x^2 + k_y^2 - k^2\right)^{\frac{1}{2}} & \text{otherwise} \end{cases}$$

The radiation condition requires that for z ≥ 0. For z < 0 -j should be replaced for +j. An imaginary kz corresponds to an evanescent PWS which is rapidly attenuated from the z=0 plane. Let us now concentrate on,

$$\underline{E}(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{A}\left(k_x,k_y\right)e^{-j\underline{k}\cdot\underline{r}}dk_x dk_y$$

At z=z$_t$ the x, y and z components of E can be expressed as,

$$\underline{E_x}(x,y,z_t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{A_x}\left(k_x,k_y\right)e^{-j\underline{k}\cdot\underline{z_t}}e^{-j\left(k_x x+k_y y\right)}dk_x dk_y$$

$$\underline{E_y}(x,y,z_t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{A_y}\left(k_x,k_y\right)e^{-j\underline{k}\cdot\underline{z_t}}e^{-j\left(k_x x+k_y y\right)}dk_x dk_y$$

$$\underline{E_z}(x,y,z_t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{A_z}\left(k_x,k_y\right)e^{-j\underline{k}\cdot\underline{z_t}}e^{-j\left(k_x x+k_y y\right)}dk_x dk_y$$

Now E$_z$ can be more easily obtained from the earlier expressions

$$\nabla\cdot\underline{E} = \nabla\cdot\underline{H} = 0$$

and

$$k_x A_x\left(k_x,k_y\right)+k_y A_y\left(k_x,k_y\right)+k_z A_z\left(k_x,k_y\right)=0$$

Now when z = 0 the expression above become,

$$\underline{E_x}(x,y,0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{A_x}\left(k_x,k_y\right)e^{-j\left(k_x x+k_y y\right)}dk_x dk_y$$

$$\underline{E_y}(x,y,0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{A_y}\left(k_x,k_y\right)^{-j\left(k_x x+k_y y\right)}dk_x dk_y$$

From the expressions above the following Fourier transforms can be derived.

$$\underline{A_x}\left(k_x,k_y\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{E_x}(x,y,0)e^{-j\left(k_x x+k_y y\right)}dk_x dk_y$$

$$\underline{A_y}\left(k_x,k_y\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{E_y}(x,y,0)^{-j\left(k_x x+k_y y\right)}dk_x dk_y$$

where these two sets of equations are Fourier transform pairs . It can be shown from these expressions that for $E_x(x, y, o)$ and $A_x(x, y, 0)$ that the x and y components are decoupled. Upon integration E(x, y, z) yields,

$$\underline{E}(x,y,z)=\frac{je^{-j\underline{k}\cdot\underline{r}}}{r}k_z\underline{A}\left(k_x,k_y\right)$$

This is since,

$$r=x\underline{x}+y\underline{y}+z\underline{z}$$

It must be noted that the phase component is a much stronger function that the amplitude function which drops off at proportion to 1/r.

This is a simple and useful relationship that exists between the far field and the PWS of the antenna under test. In this expression k_z is always real because an imaginary kz component douse not propagate to the far field zone. Therefore, for this equation we have,

$$k_x=k\sin\theta\cos\phi$$

$$k_y=k\sin\theta\sin\phi$$

$$k_z=k\cos\theta$$

The UV space system has been adopted within the DRA NFS software system. The field probe is assumed to be an ideal point dipole. This is consistent with the theory as described above. In practice the probes are both directive and suffer from multi path effects as a result from multiple reflections between the antenna and the probe. Probe correction factors need to be employed in the far field transformation process in order to correct for this. These probe correction factors are implemented with in the NFS facility.

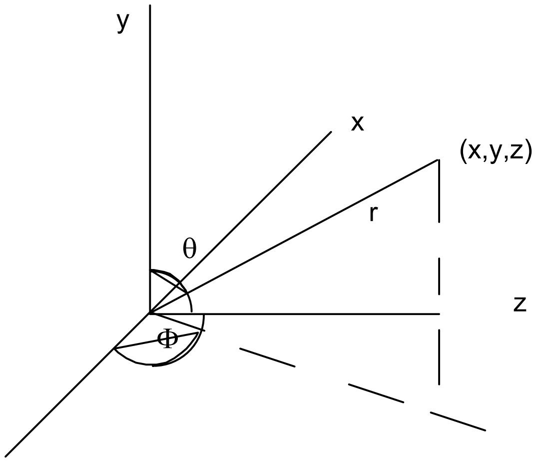

Figure A2